

Math 10B - Calculus of Several Variables II

Quiz 1

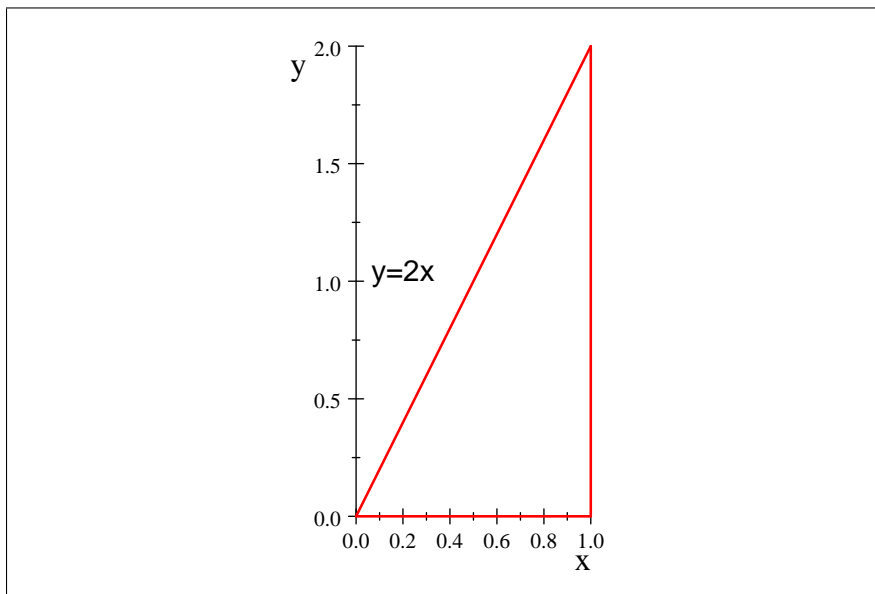
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Name: _____

Evaluate the following integral:

$$\int_0^2 \int_{\frac{y}{2}}^1 e^{-x^2} dx dy.$$

Solution. First, since we cannot do the integral $\int e^{-x^2} dx$, we need to change the order of integration. So let's look at the region of integration:



Now we can easily change the order of integration:

$$\begin{aligned} \int_0^2 \int_{\frac{y}{2}}^1 e^{-x^2} dx dy &= \int_0^1 \int_0^{2x} e^{-x^2} dy dx \\ &= \int_0^1 2x e^{-x^2} dx \stackrel{u=x^2}{=} \int_0^1 e^{-u} du \\ &= -e^{-u} \Big|_0^1 = -e^{-1} + e^0 \\ &= 1 - \frac{1}{e} = \frac{e-1}{e}. \end{aligned}$$

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